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## Describing Sequential Systems

- So far we have used Truth Tables to describe sequential systems
- Can also use Bubble Diagrams and Algorithmic State Machine Charts (ASM) to describe a sequential system.
- Another name for a sequential system is a Finite State Machine (FSM).
- A sequential system with N flip-Flop has $2^{\mathrm{N}}$ possible states, so the number of possible states is FINITE.
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## DFF as a Finite State Machine

## A DFF is a finite state machine with two possible states

 Lets call these states S0 and S1. (state enumeration).Furthermore, lets say when the $\mathbf{Q}$ output $=$ ' 0 ', then we are in State $S 0$, and that when $Q$ output $=' 1$ ', we are in State S1. This is called the State Encoding.


Bubble Diagram: States represented by bubbles. State transitions represented by arrows. Labeling on arrows represent input values (in this case, the D-input!).
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$\qquad$ Labeling inside bubbles represent output values.

Algorithmic State Machine Chart for DFF
(SO)


A Finite State Machine (FSM) can be described via either a Bubble diagram or an ASM chart.

ASM charts are better for complex FSMs. We will use ASM charts in this class.

State S 0 is usually the asynchronous Reset state.

## Algorithmic State Chart (ASM)

- An ASM chart can be used to describe FSM behavior

Only three action signals can appear within an ASM chart:

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## Algorithmic State Machine Chart for JKFF

(s)


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## Finite State Machine Implementation

Given an Algorithmic State Machine chart that describes a Finite State Machine, how do we implement it?????

Step \#1: Decide on the State Encoding (how many Flip Flips do I use and how what should the FF outputs be for EACH state). The problem definition may decide the state encoding for you.
Step \#2: Decide what kind of FFs to use! (We will always
$\qquad$ use DFFs in this class, but you could use JKFFs or TFFs if you wanted to).
Step \#3: Write the State Transition Table.
Step \#4: Write the FF input equations, and general output equations from the state transistion table.
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## Problem Definition

Design a Modulo three counter. The count sequence is:

$$
" 00 " \rightarrow " 01 " \rightarrow " 10 " \rightarrow " 00 " \rightarrow " 01 " \rightarrow " 10 ", \text { etc. }
$$

There is an "en" input that should control counting (count when en=1, hold value when en=0). Assume ACLR line used to reset counter to " $\mathbf{0 0}$ ".

How many states do we need? Well, we have three unique output values, so lets go with three states.

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ASM Chart for Modulo Three Counter

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State Transition Table $\qquad$
$\qquad$ present state, input values.

| Inputs(EN) | Present State | Next State |  |
| :--- | :--- | :--- | :--- |
| $\mathbf{y}$ | S0 | S0 | $\mathbf{0 0}$ |
| 0 | S1 | S1 | 01 |
| 0 | S2 | S2 | 10 |
| 1 | S0 | S1 | 00 |
| 1 | S1 | S2 | 01 |
| 1 | S2 | S0 | 10 |

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## Decisions

- State encoding - will be based on number of FFs we use.
- Three states means the minimum number of FFs we can use two $\mathrm{FFs}\left(\log _{2}(3)=2\right)$.
- If we use two FFs, then could pick a state encodings like:
- S0: 00, S1: 01, S2: 10 (binary counting order)
- S0: 01, S1:01, S2: 11 (gray code - may result in less combinational logic)
- Could also use 1 FF per state ( 3 FFs ) and use one hot encoding
- S0:001, S1: 010, S2: 100 (may result in less combinational logic) ${ }_{\text {BR } 8999}$ 12
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## Decisions (cont.)

- What type of FF to use?
- DFF - most common type, always available in programmable logic
- JKFF - sometimes available, will usually result in less combinational logic (more complex FF means less combinational logic external to FF)

Lets use two FFs with state encoding $\mathbf{S 0 = 0 0 , S 1 = 0 1 \text { , }}$ S2=10.
Lets use DFFs.

## New State Transition Table

Modify State Transition table to show what FF inputs need to be in order to get to that state. Also, use actual state encodings

| Inputs(EN) | Present <br> State <br> $($ Q1Q0 $)$ | Next <br> State <br> $(\text { Q1Q })^{*}$ | D1D0 | Y |
| :--- | :---: | :---: | :---: | :---: |
| $\mathbf{0}$ | 00 | 00 | 00 | 00 |
| 0 | 01 | 01 | 01 | 01 |
| 0 | 10 | 10 | 10 | 10 |
| 1 | 00 | 01 | 01 | 00 |
| 1 | 01 | 10 | 10 | 01 |
| 1 | 10 | 00 | 00 | 10 |

For DFFs, D inputs are simply equal to next state!!!!

## D-input Equations, Y equations

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ized equations:

$$
\begin{aligned}
& \text { D0 = EN' } \mathbf{Q 1}^{\prime} \mathbf{Q} \mathbf{Q} \text { + EN Q1'Q0' } \\
& \text { D1 = EN' } \mathbf{Q 1} \mathbf{Q} 0 \text { ' }+ \text { EN Q1' } \mathbf{Q} 0
\end{aligned}
$$

$\mathbf{Y 0}=\mathbf{Q} 0$
$\mathbf{Y 1}=\mathbf{Q} 1$
The output $Y$ is simply the DFF outputs! Here is one case where state encoding is affected by problem definition (does not make much sense to use a different state encoding, even though we could do it).

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| What if we used JKFFs? |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Need to change State Transistion table to reflect JK input values. |  |  |  |  |  |
| Inputs EN | Present State (Q1Q0) | Next State (Q1Q0)* | J1 K1 | J0 K0 | Y |
| 0 | 00 | 00 | 0 X | 0 X | 00 |
| 0 | 01 | 01 | 0 X | x 0 | 01 |
| 0 | 10 | 10 | X 0 | 0 X | 10 |
| 1 | 00 | 01 | 0 X | 1 X | 00 |
| 1 | 01 | 10 | 1 X | X 1 | 01 |
| 1 | 10 | 00 | X 1 | 0 X | 10 |

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JK FF Q transitions: $\mathbf{0} \rightarrow \mathbf{0}(\mathrm{J}=\mathbf{0}, \mathrm{K}=\mathrm{X}) ; \mathbf{0} \rightarrow \mathbf{1}(\mathrm{J}=\mathbf{1 , K}=\mathrm{X})$; $\qquad$ $\mathbf{1} \rightarrow \mathbf{1}(\mathrm{J}=\mathrm{X}, \mathrm{K}=\mathbf{0}) ; \mathbf{1} \rightarrow \mathbf{0}(\mathrm{J}=\mathrm{X}, \mathrm{K}=\mathbf{1}) ;$ BR $8 / 99$

JK Input Equations, Output Equations $\qquad$

Unoptimized equations

| $\mathbf{J 0}=\mathbf{E N}$ Q1' Q0, | K0 $=$ EN Q1' Q0 |
| :--- | :--- |
| $\mathbf{J 1}=\mathbf{E N}$ Q1' Q0 | K1 $=$ EN Q1 Q0, |
|  |  |
| $\mathbf{Y 0}=\mathbf{Q 0}$ |  |
| $\mathbf{Y 1}=\mathbf{Q 1}$ |  |

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$\mathrm{Y} 1=\mathbf{Q 1}$
Using JK FFs will mean simpler external optimized combinational logic because FFs are more complex $\qquad$ (provide more functionality).
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| 3 DFFs and One Hot Encoding State encoding: $\mathbf{S 0}=\mathbf{0 0 1}, \mathbf{S 1}=\mathbf{0 1 0}, \mathbf{S} 2=100$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \text { Inputs } \\ & \text { EN } \end{aligned}$ | $\begin{aligned} & \text { Present } \\ & \text { State } \\ & \text { (Q2Q1Q0) } \end{aligned}$ | $\begin{gathered} \text { Next } \\ \text { State } \\ (\text { Q2Q10 } 0 \text { ) } \end{gathered}$ | D2D1D0 | Y |  |
| 0 | 001 | 001 | 001 | 00 |  |
| 0 | 010 | 010 | 010 | 01 |  |
| 0 | 100 | 100 | 100 | 10 |  |
| 1 | 001 | 010 | 010 | 00 |  |
| 1 | 010 | 100 | 100 | 01 |  |
| 1 | 100 | 001 | 001 | 10 |  |
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DFF input equations, Output Equations $\qquad$
D0 $=$ EN'Q0 + ENQ2
D1 = EN'Q1 + ENQ0
D2 $=$ EN'Q2 + ENQ1
$\mathbf{Y 0}=\mathbf{E N} \mathbf{Q P}^{\prime}+\mathbf{E N} \mathbf{Q 1}=\mathbf{Q 1}$
$\mathbf{Y} 1=\mathbf{E N}^{\prime} \mathbf{Q}^{2}+\mathbf{E N} \mathbf{Q}^{2}=\mathbf{Q}^{2}$
In equations, because a FF $Q$ will only be ' 1 ' in a single state, do not have to include all FFs to define state!! ( Q2'Q1'Q0 = Q0!!, Q2'Q1Q0' = Q1!, Q2Q1'Q0' = Q2!!) This is one of the advantages of one-hot encoding!
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Generic Next State Equations
Generic next state equations can be written directly from the ASM
chart as an alternative to the Transition table
$\mathrm{S}^{*}=$ (conditions to remain in this state) + (conditions to enter state)
From ASM chart of modulo three counter:
S0* = EN' S0 + EN S2
S1* = EN' S1 + EN S0
S2* = EN'S2 + EN S1
If One hot encoding and DFFs are used, then Generic Next
State equations ARE the specific next State Equations!!
D0 $=$ EN'Q0 + EN Q2
D1 $=$ EN'Q1 + EN Q0
D2 $=$ EN' Q2 + EN Q1

